STEVENS
Institute of Technology

Application of Channel Estimation to Underwater Acoustic Communication

Brian S. Borowski

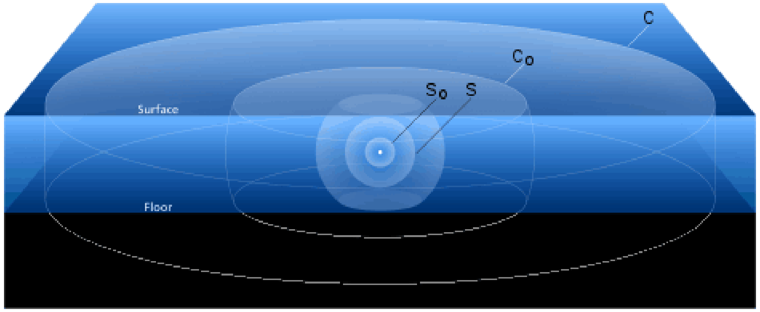
Thesis Advisory Committee:
- Dr. Dan Duchamp (Chairman), Stevens Institute of Technology (CS)
- Prof. Bruce McNair, Stevens Institute of Technology (ECE)
- Dr. Charles Suffel, Stevens Institute of Technology (Math)
- Dr. Joseph Mitola III, Stevens Institute of Technology (Provost Office)

This presentation includes:

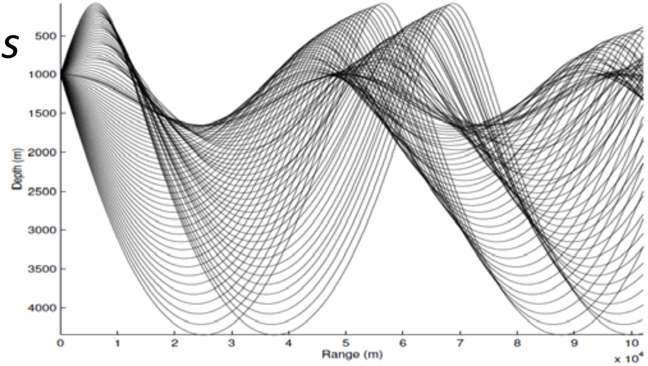
- Introduction
- Thesis statement
- Contributions to the field in:
  - Channel characterization
  - Channel/PHY simulation
  - Adaptive software modem
- Summary

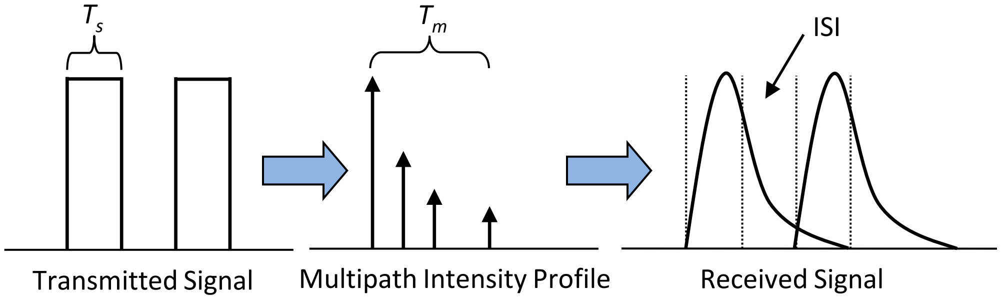
Difficulties with Underwater Acoustic Communication

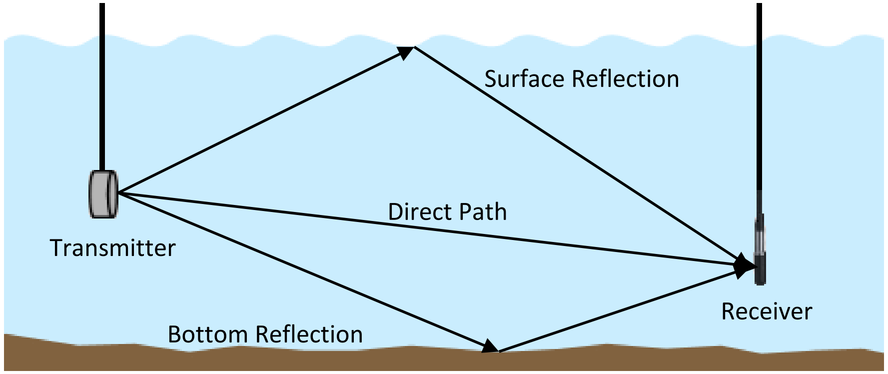
- Transmission loss
- Stratification
- Noise
- Doppler spread
- High propagation delay
- Multipath propagation



shadow zones







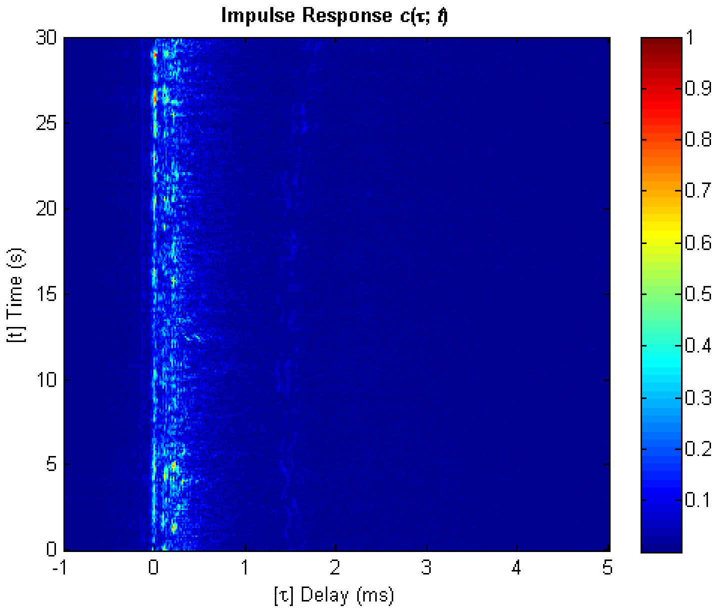
Thesis Statement

Channel estimation techniques can be employed in both a network simulator and software modem to quantify the channel-induced distortion of acoustic signals and thereby to improve the quality of simulation and adaptability of modulation and demodulation, respectively.

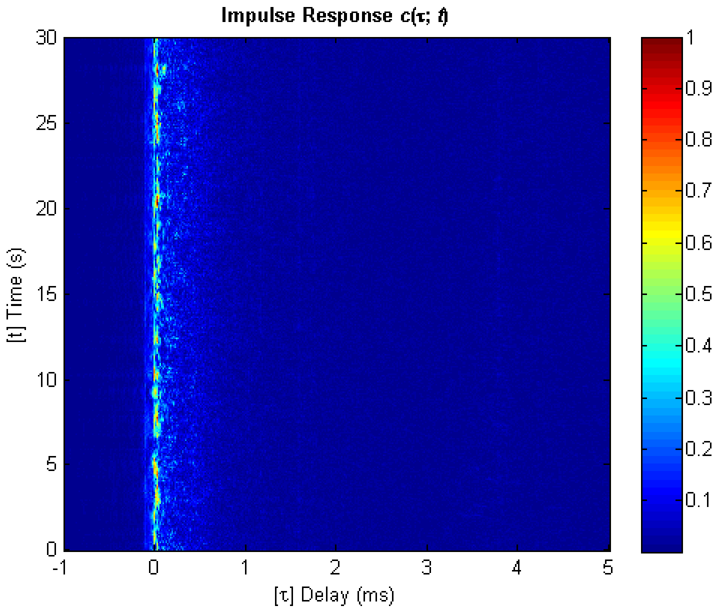
Why Do Channel Characterization?

- Relatively few papers have focused on the fundamental process of characterizing the underwater acoustic channel
- There is no typical underwater channel
- Is a necessary step for the design of a successful communication system
- Numerous channel measurements are required to build up a database of underwater environments for more realistic network simulations

Time-Variant Impulse Response



Successive time-variant impulse response estimates at 505m

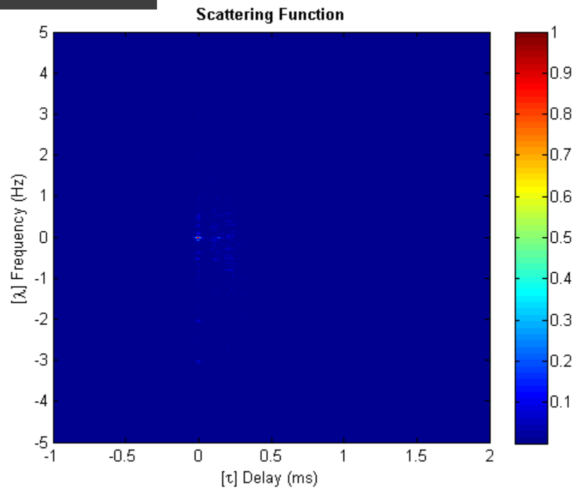


Successive time-variant impulse response estimates at 200m

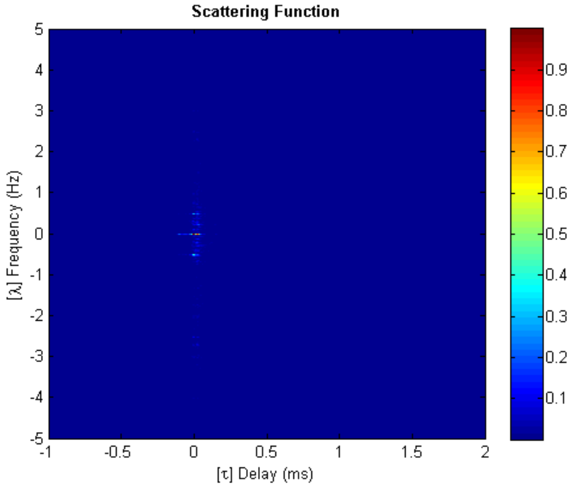
Scattering Function

- Gives the average power output of the channel as a function of time delay τ and Doppler frequency λ
- Is the basis for computing the remainder of the channel characterization functions

$$S_c(\tau;\ \lambda) = \int_{-\infty}^{\infty} A_c(\tau;\ \Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t$$



Scattering function at 505m



Scattering function at 200m

Multipath Intensity Profile

- $P(\tau)$ gives the average power output as a function of time delay τ
- Computed by summing the power levels over the λ values

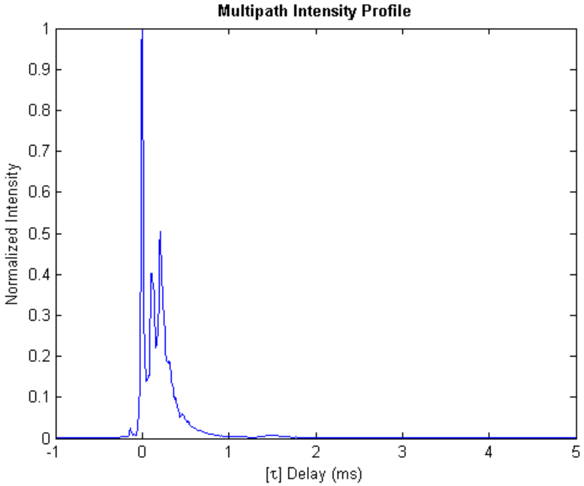
$$P(\tau) = \int S_c(\tau;\, \lambda)\, d\lambda$$

Doppler Shift and Spread (Hz) of Strong Multipath Arrivals

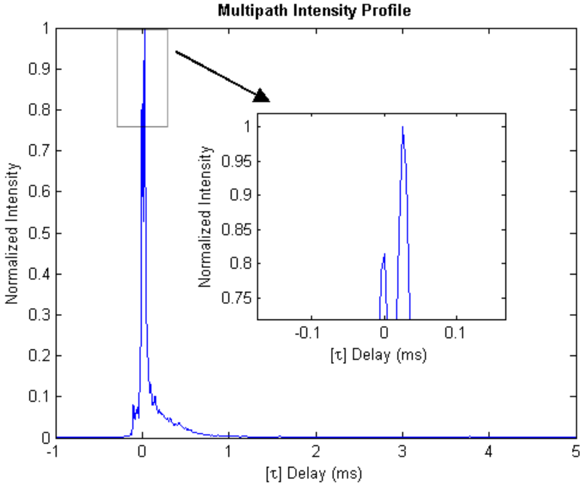
	200m				505m			
	Time (ms)	Intensity	Shift	Spread	Time (ms)	Intensity	Shift	Spread
Arrival 1	0.000	0.8136	-0.1945	2.6790	0.000	1.0000	-0.3642	2.8315
Arrival 2	0.025	1.0000	-0.2588	2.6948	0.105	0.4033	-0.3667	3.0616
Arrival 3	–	–	–	–	0.205	0.5041	-0.4556	3.0057

Delay Spread of Multipath Intensity Profile (ms)

	Mean Excess Delay	RMS Delay Spread	Maximum Excess Delay (-10 dB)
200m	0.0907	0. 1478	0.1800
505m	0.1789	0.1636	0.4150



Multipath intensity profile at 505m



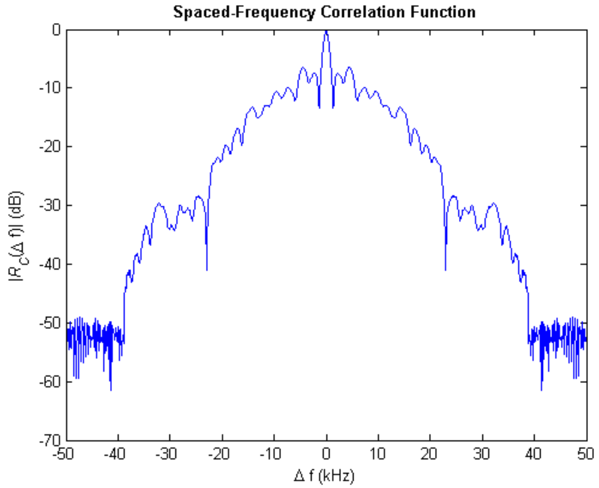
Multipath intensity profile at 200m

Spaced-Frequency Correlation Function

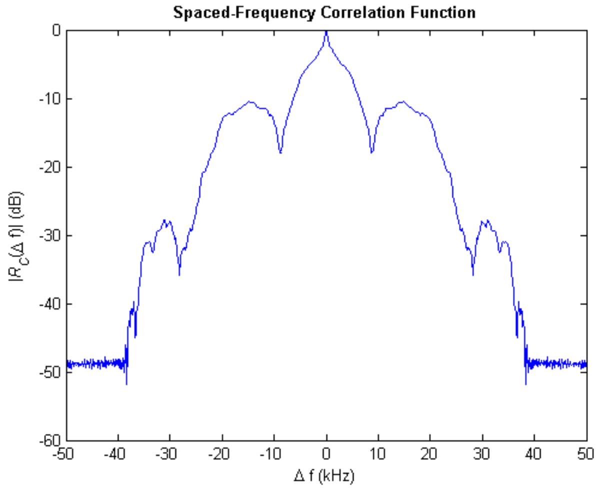
- Fourier transform of the MIP
- Indicates the coherence bandwidth of the channel, a statistical measure of the range of frequencies over which the channel passes all spectral components with approximately equal gain and linear phase

Coherence Bandwidth (Hz)

	-3 dB	-6 dB	-10 dB
200m	2331	8160	12490
505m	1166	1665	2165



Spaced-frequency correlation function at 505m



Spaced-frequency correlation function at 200m

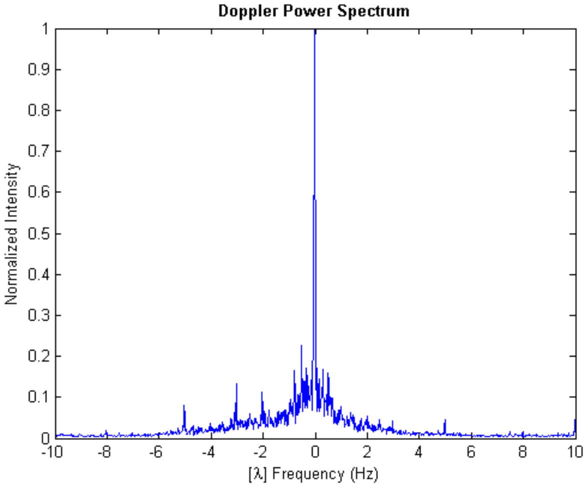
Doppler Power Spectrum

- Provides the signal intensity as a function of the Doppler frequency λ
- Computed by summing the power of spectral components of the scattering function over the time delay τ

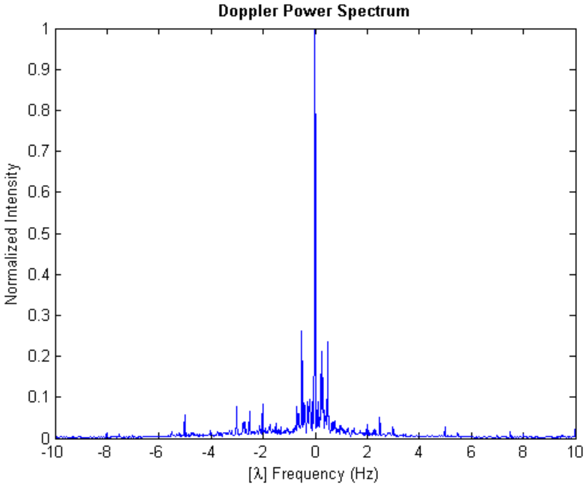
$$P(\lambda) = \int S_c(\tau;\, \lambda)d\tau$$

Overall Doppler Shift and Spread (Hz)

	Shift	Spread
200m	-0.2357	3.3231
505m	-0.3381	3.3843



Doppler power spectrum at 505m



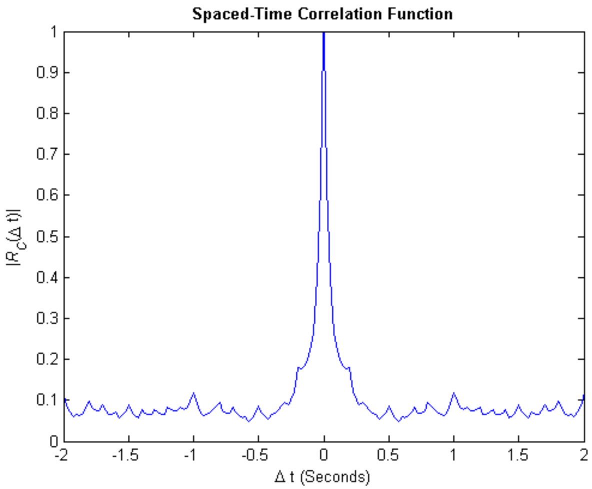
Doppler power spectrum at 200m

Spaced-Time Correlation Function

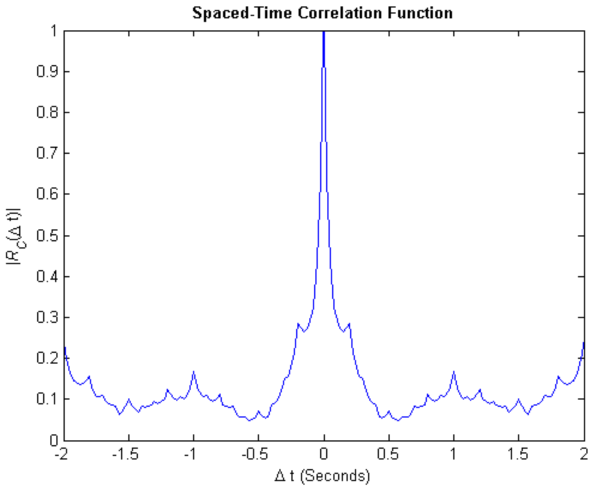
- Fourier transform of the Doppler power spectrum
- Provides the channel's coherence time, a measure of the expected time duration over which the channel's response is essentially invariant

Coherence Time (ms)

	0.5 (-3dB)	0.25 (-6dB)	0.1 (-10 dB)
200m	50	400	699
505m	50	150	500



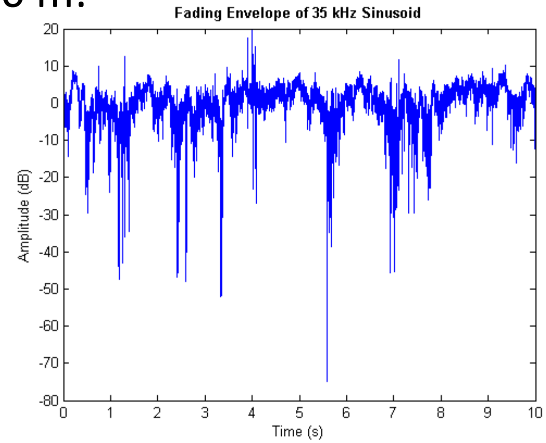
Spaced-time correlation function at 505m

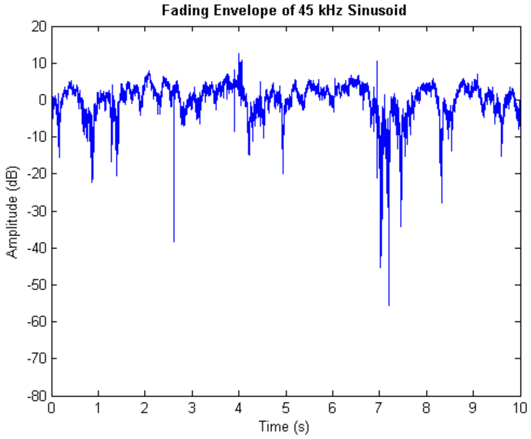


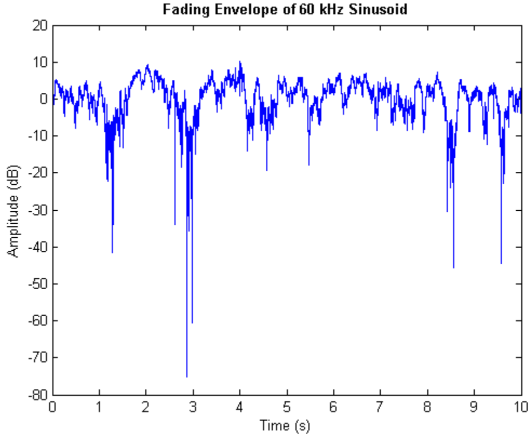
Spaced-time correlation function at 200m

Fading Characteristics

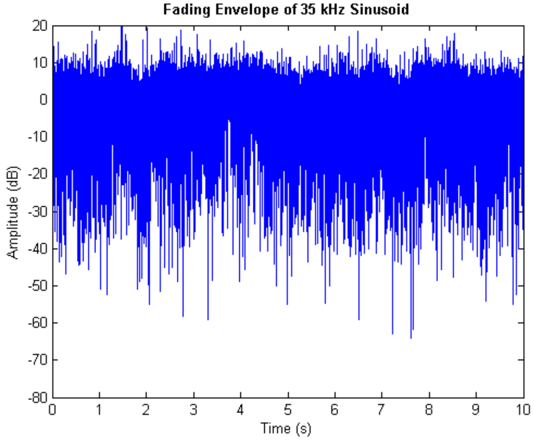
200 m:

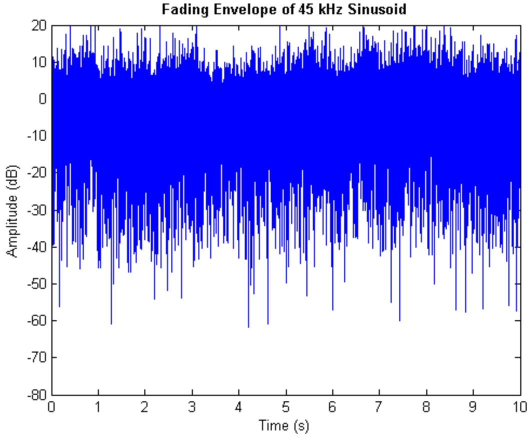

Fading Envelope of 35 kHz Sinusoid

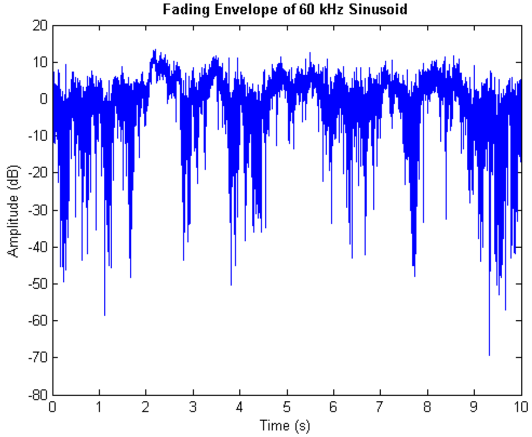

Fading Envelope of 45 kHz Sinusoid


Fading Envelope of 60 kHz Sinusoid

505 m:


Fading Envelope of 35 kHz Sinusoid


Fading Envelope of 45 kHz Sinusoid


Fading Envelope of 60 kHz Sinusoid

Distribution Fitting

- Maximum likelihood estimation was used to fit the data to the Rayleigh, Rice, and Nakagami-*m* (as well as other less likely) distributions

- Kullback-Leibler divergence, D_{KL}

$$D_{KL}(P \parallel Q) = \sum_i P(i) log_2 \frac{P(i)}{Q(i)}$$

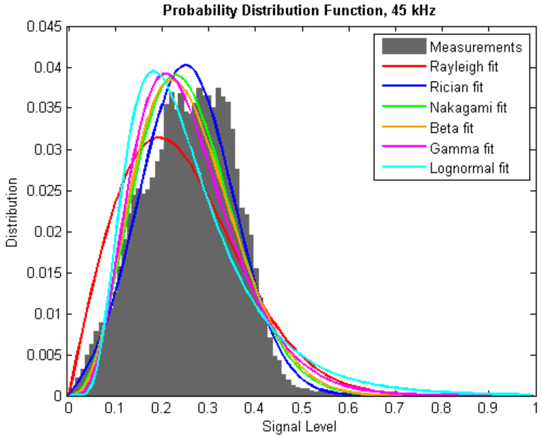
- Bhattacharyya distance, D_B

$$D_B(P,Q) = -log_2(BC) \text{ where } BC = \sum_i \sqrt{P(i)Q(i)}$$

- Metric based on the Bhattacharyya coefficient, proposed by Comaniciu, Ramesh, and Meer, D_{CRM}

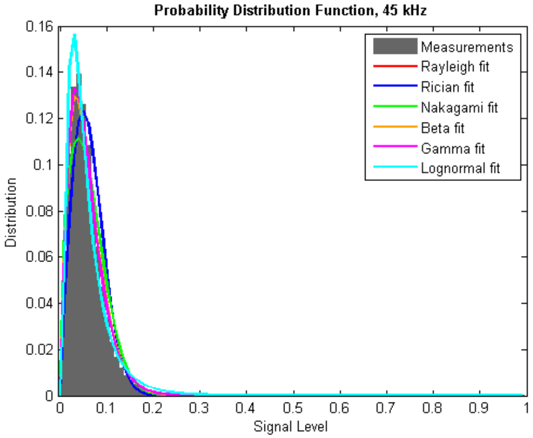
$$D_{CRM}(P,Q) = \sqrt{1 - BC} \text{ where } BC = \sum_i \sqrt{P(i)Q(i)}$$

Distribution Fitting Results



200m

	D_{KL}	D_B	D_{CRM}	Parameters
Beta	0.0771	0.0189	0.1142	[alpha = 4.2516, beta = 12.4813]
Gamma	0.1273	0.0325	0.1492	[alpha = 5.3128, beta = 0.0481]
Lognormal	0.2731	0.0653	0.2104	[mu = -1.4621, sigma = 0.4930]
Nakagami-m	0.0582	0.0147	0.1008	[m = 1.6674, omega = 0.0746]
Rayleigh	0.1547	0.0447	0.1746	[sigma = 0.1932]
Rice	0.0232	0.0058	0.0632	[s = 0.2300, sigma = 0.1042, K = 2.4372]



505m

	D_{KL}	D_B	D_{CRM}	Parameters
Beta	0.0727	0.0089	0.0782	[alpha = 2.2509, beta = 36.9124]
Gamma	0.0689	0.0081	0.0747	[alpha = 2.4133, beta = 0.0237]
Lognormal	0.4863	0.0138	0.0975	[mu = -3.0819, sigma = 0.7095]
Nakagami-m	0.0939	0.0127	0.0936	[m = 0.7174, omega = 0.0049]
Rayleigh	0.1947	0.0240	0.1284	[sigma = 0.0494]
Rice	0.1947	0.0240	0.1284	[s = 0.0000, sigma = 0.0494, K = 0.0000]

Implications for Communication

- (Time domain) If $T_m > T_s$, the channel exhibits frequency-selective fading, which results in channel-induced ISI
  - At 200m, T_m = 0.1800 ms => 5555 symbols per second
  - At 505m, T_m = 0.4150 ms => 2410 symbols per second
- (Frequency domain) If $W > f$, where W is the bandwidth required for modulation and f is the coherence bandwidth, the channel imposes frequency-selective degradation
- (Time domain) If $T_c > T_s$, the channel exhibits slow fading
  - In the Hudson, the -3dB coherence time is 50ms, which is most likely significantly longer than T_s => slow fading channel
- (Frequency domain) If $W > f_d$, the channel is referred to as slow fading
- Harsh condition over long links => deploy spread spectrum, CDMA, multi-hop network

Summary and Future Work

- Summary
  - LFM chirp signals and a comb signal were emitted during the experiment
  - Environmental conditions were recorded
  - Impulse response estimates were used to derive channel characterization functions
  - Various distributions were fitted to amplitude fluctuations
- Future Work
  - Sound the channel over other distances and at different times
  - Permanently affix channel sounding equipment to buoys or river floor for taking year-round measurements

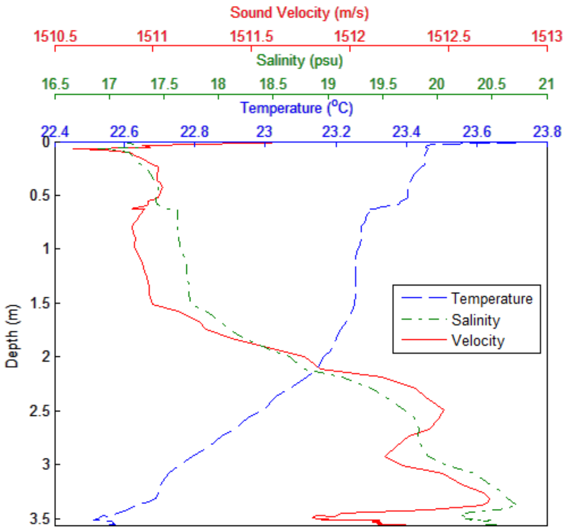
Current simulations…

- Use empirical formula to describe the underwater acoustic channel [ns-2 extension, Harris 2007]
  - Thorp's approximation for the absorption of a wave at a given frequency
  - Formulas for power spectral density of noise-producing agents
- Use BELLHOP to model to compute SINR values for which bit error rates can be looked up [World Ocean Simulation System, Guerra 2009]
- Focus only on power consumption and propagation delay/range [Aqua-Sim 2010]

My solution…

- Is based fully on measurements
- Converts real application-generated packets into acoustic waveforms
- Mixes acoustic waveforms with the channel to provide accurate bit error rates for packets passing through channels that change slowly
- Can simulate any channel for which measurements exist
- Is modular and can be easily extended with new processing blocks or alternate implementations

Propagation (D_P) and Transmission (D_T) Delays



Sound velocity profile for 505-meter channel

D_P = link distance / sound velocity

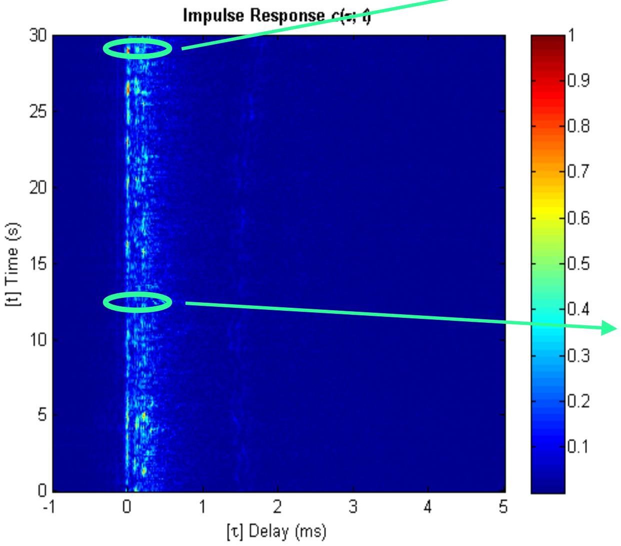
D_T = number of symbols / symbols per second

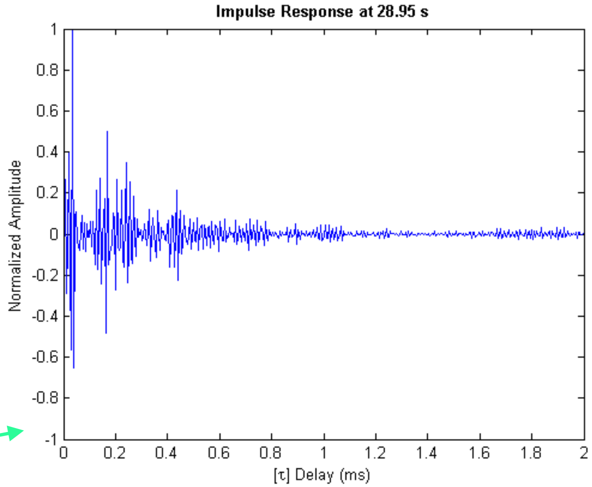
Transmission Loss

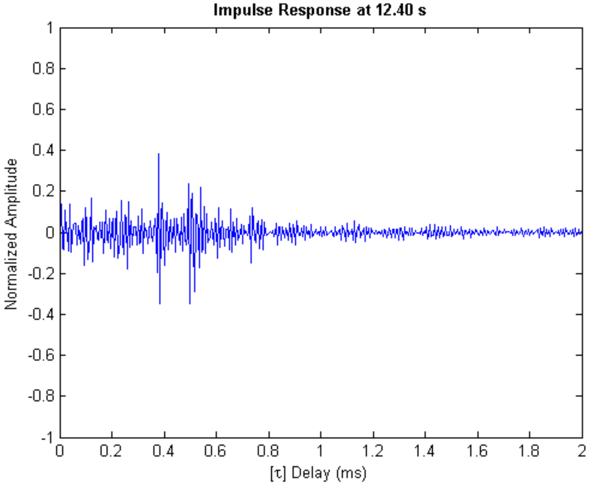
$$TL = 10\log(r) + \alpha r,\quad \alpha = 0.058\ \text{dB/m}$$

Impulse Response

Successive time-variant impulse response estimates of Hudson at 505 meters

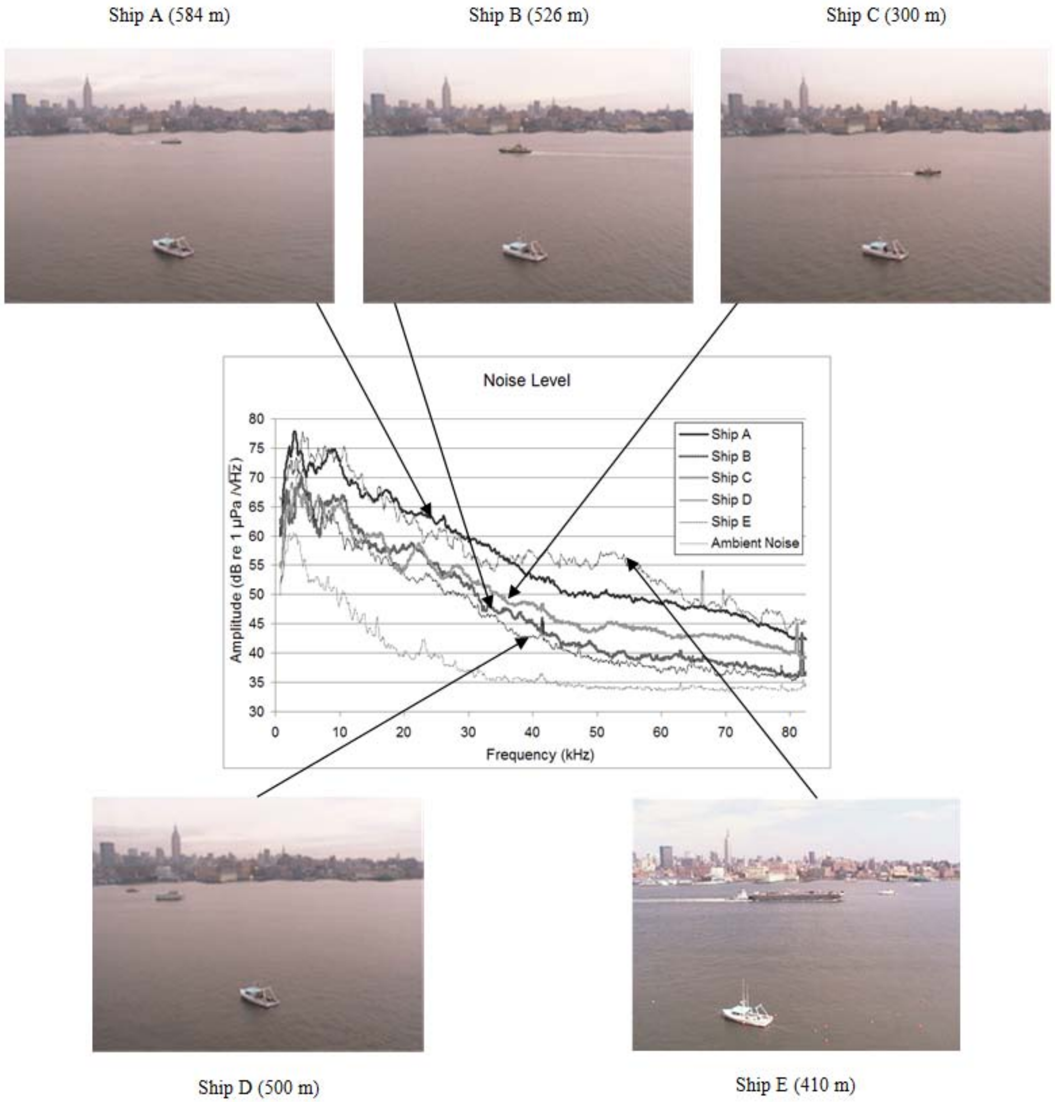






Noise

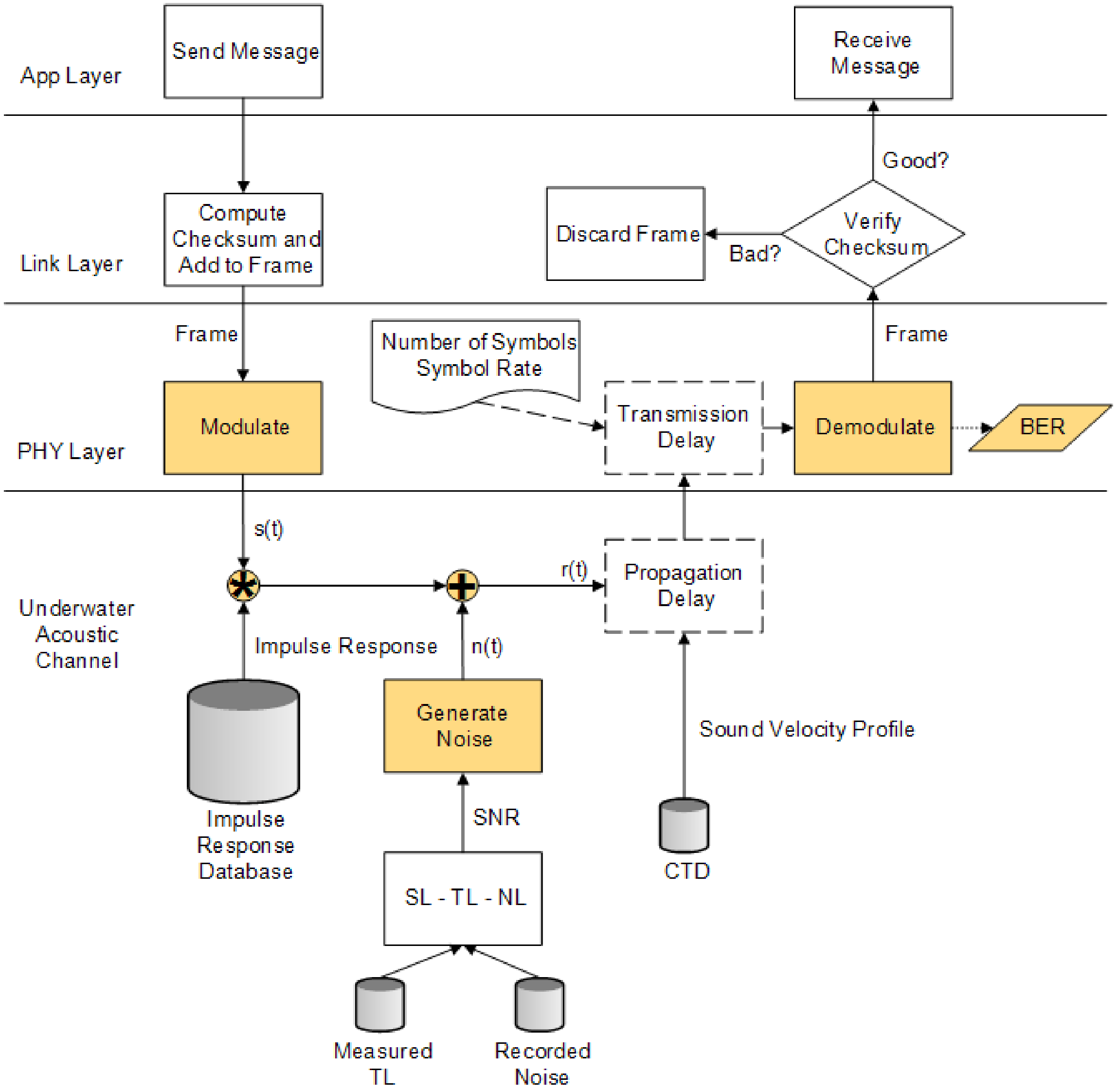
Noise levels in the Hudson River estuary produced by different passing ships



Simulation Architecture

Architecture of OMNeT++ simulation for PHY layer and underwater acoustic channel.

- Areas in gray represent data from measurements.
- Areas in yellow represent sections implemented in MATLAB.
- A simple implementation of the application and link layers is included for future extensions but is not the focus of this work.



Validation

Overall comparison of BERs obtained with data transmission versus convolution

Overall % Difference			
Average	Min	Max	Std. Dev.
3.34	0.00	33.31	8.21

Comparison of BERs obtained with data transmission versus convolution, per carrier frequency

			% Difference			
Modulation	Demodulation	Implementation	Average	Min	Max	Std. Dev.
FSK	Envelope Detector	Amplitude Comp.	2.60	0.00	20.16	5.22
		Hard Limiter	3.34	0.00	27.69	7.28
	Quadrature Receiver	Default	4.35	0.00	24.41	7.36
		Hard Limiter	2.89	0.00	24.31	6.14
PSK	Correlator	N/A	3.51	0.00	33.31	9.01

Comparison of BERs obtained with data transmission versus convolution, grouped by the type of modulation-demodulation

	% Difference			
Frequency (Hz)	Average	Min	Max	Std. Dev.
7,500	1.74	0.00	12.47	3.35
12,500	3.66	0.00	33.31	8.16
17,500	4.61	0.00	27.69	8.11

Comparison of BERs obtained with data transmission versus convolution, grouped by bit rate

	% Difference			
Bit Rate (bps)	Average	Min	Max	Std. Dev.
250	0.00	0.00	0.00	0.00
500	2.03	0.00	24.41	6.29
1250	7.50	0.00	27.69	9.72
2500	3.91	0.00	13.25	4.21
>3000	3.25	0.00	33.31	8.41

Simulation Output

```
Generating datagram 2...
Bits sent:
10001010:01010100:00000000:10111110:00000000:00000000:00000000:00000001
01010100:01101000:01100101:00100000:01010101:00101110:01010011:00101110
00100000:01001000:01101111:01110101:01110011:01100101:00100000:01101111
01100110:00100000:01010010:01100101:01110000:01110010:01100101:01110011
01100101:01101110:01110100:01100001:01110100:01101001:01110110:01100101
01110011:00100000:01110110:01101111:01110100:01100101:01100100:00100000
01111001:01100101:01110011:01110100:01100101:01110010:01100100:01100001
01111001:00100000:01110100:01101111:00100000:01100001:01100100:01101101
01101111:01101110:01101001:01110011:01101000:00100000:01010111:01101001
01101100:01110011:01101111:01101110:00100000:01101111:01110110:01100101
01110010:00100000:01110100:01101000:01100101:00100000:01100011:01101111
01101101:01101101:01100101:01101110:01110100:00101110:00100000:01001100
01101111:01110101:01110000:01100001:01110011:01110011:01101001:00100000
01110011:01100001:01101001:01100100:00100000:01110011:01101001:01101101
01101001:01101100:01100001:01110010:00100000:01100010:01100101:01101000
01100001:01110110:01101001:01101111:01110010:00100000:01110111:01101111
01110101:01101100:01100100:00100000:01100010:01100101:00100000:01110111
01101001:01101100:01100100:01101100:01111001:00100000:01110101:01101110
01100001:01100011:01100011:01100101:01110000:01110100:01100001:01100010
01101100:01100101:00100000:01101001:01101110:00100000:01110100:01101000
01100101:00100000:01010110:01101001:01110010:01100111:01101001:01101110
01101001:01100001:00100000:01001000:01101111:01110101:01110011:01100101
00100000:01101111:01100110:00100000:01000100:01100101:01101100:01100101
01100111:01100001:01110100:01100101:01110011:00101110:
PSK-modulated data in 166800 samples.
Source level         : 120.00 dB
Transmission loss    : 56.32 dB
Noise level          : 33.82 dB
SNR                  : 29.86 dB
Opening IR file data/IR_505m/IR_277.wav.
Number of samples in packet after convolution: 176400
```

```
Demodulating 190 bytes.
BER: 4.67%
Bits received:
10001010:01010100:00000000:10111110:00000000:00000000:00000000:00000001
01010110:01101000:01110101:00100000:01010101:00101111:01010011:00101111
00100000:01001000:01101111:01110101:01110011:01100101:00100000:01101111
01110111:00110000:01010010:01110101:01110000:01110010:01110101:01110011
01100101:00101111:00110100:00110001:01110100:01101001:01110111:01110101
00110011:00100000:01110111:01101111:01110100:00110101:01100100:00100000
01111001:01100101:01110011:01110100:00100101:01110010:01110100:00100001
01111001:00100000:01110100:01101111:00100000:01100001:01100100:01101101
01101111:01101110:00101001:01110011:01101000:00100000:01010111:01101001
01101100:01110011:01101111:01101110:00100000:00101111:01110110:00110101
01110010:00100000:01110100:01101000:01110101:00100000:01100011:01101111
01101101:01101101:01100101:00101110:00110100:00101111:00100000:01001100
00101111:01110101:01110000:01100001:01110011:01110011:01101001:00100000
01110011:01100001:01101001:01100100:00100000:00110011:01101001:01101101
01101001:01101100:00100001:01110010:00100000:01100011:01110101:00101000
01100001:01110111:00101001:01101111:01110010:00100000:00110011:01101111
01110101:00101100:00100100:00100000:01100011:01110101:10100000:01110111
01101001:01101100:01110100:00101100:00111001:00100000:01110101:01101111
00110001:01100011:01100011:01100101:00110000:01110100:00110001:01100011
01101100:00100101:00100000:01101001:01101111:00100000:01110100:00101000
01100101:00100000:01010110:01101001:01110010:01110111:00101001:01101110
00101001:01100001:00100000:01001000:01101111:01110101:01111001:01100101
00100000:01101111:01110111:00110000:01000100:00100101:01101110:01110101
01100111:01100001:01110100:00100101:01110011:00101110:
Expected payload: The U.S. House of Representatives voted yesterday to
admonish Wilson over the comment. Loupassi said similar behavior would
be wildly unacceptable in the Virginia House of Delegates.
Actual payload  : Vhu U/S/ House ow0Rupruse/41tiwu3 wot5d yest%rt!y to
admon)sh Wilson /v5r thu comme.4/ L/upassi said 3imil!r cu(aw)or 3ou,$
cu.wilt,9 uo1cce0t1cl% io t(e Virw)n)a Houye ow0D%nugat%s.
```

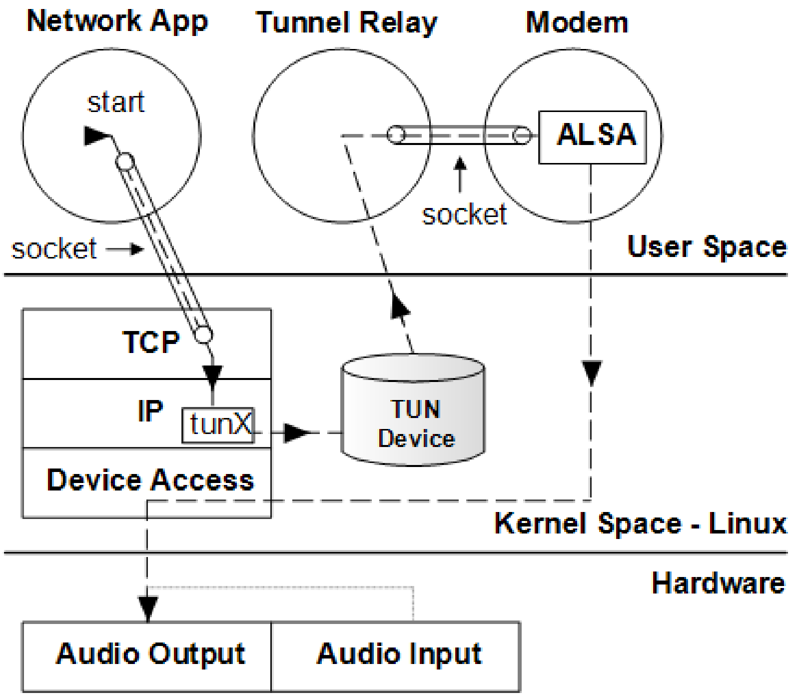
Summary and Future Work

- Summary
  - Simulation based on channel measurements that produces accurate bit error rates of application-generated packets
  - For channels that change relatively slowly, the simulation produces bit error rates that are within 3.34% of the actual bit error rates
- Future Work
  - Evaluate single impulse response model in time-variant channel
    - Utilize two dimensional convolution
    - Account for multipath fading
  - Add more types of modulation techniques
  - Implement higher layers in the network stack

- Goals
  - Easy deployment of applications written with sockets
  - Extensible platform for real-time channel estimation and communication
  - Low cost underwater acoustic modem fully implemented in software
- Features
  - Uses sound card of PC
  - Supports binary and 4-FSK (frequency shift keying) modulation
  - User-adjustable parameters, including
    - Bit rate
    - Carrier frequency
    - Detection threshold
  - Exploits a per-frame LFM (linear frequency modulated) chirp signal for synchronization and channel estimation
  - Can use Levinson-Durbin matrix inversion for equalization of slowly varying channels (zero forcing equalizer)
  - Can employ Reed-Solomon codes for error correction
  - Incoming frames and impulse response estimates can be saved to .wav and .csv files for offline analysis; SNR is computed and logged

System Architecture

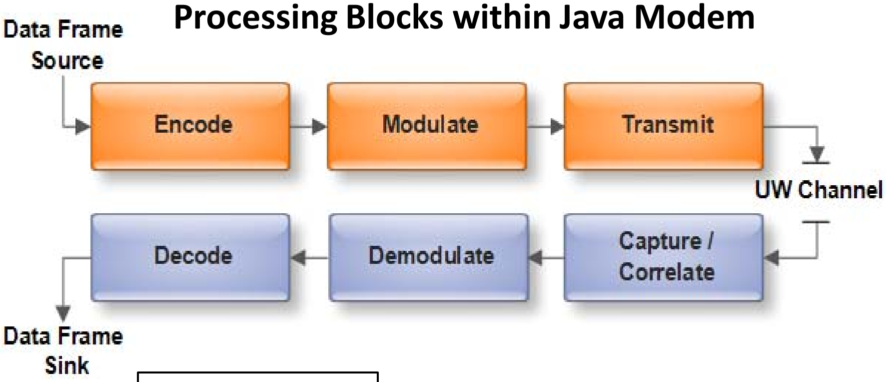
- Includes 3 levels of user space applications
  - Network app (TCP/UDP, any language)
  - Acoustic modem (Java)
  - Tunnel relay app for passing IP datagrams from the network app to and from the software modem (C)

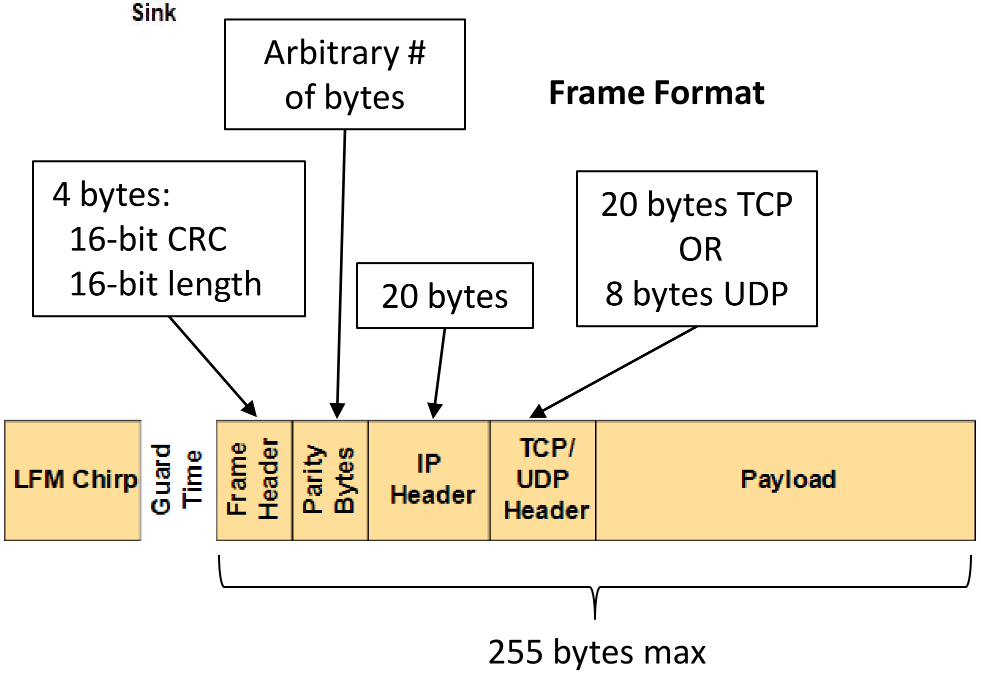


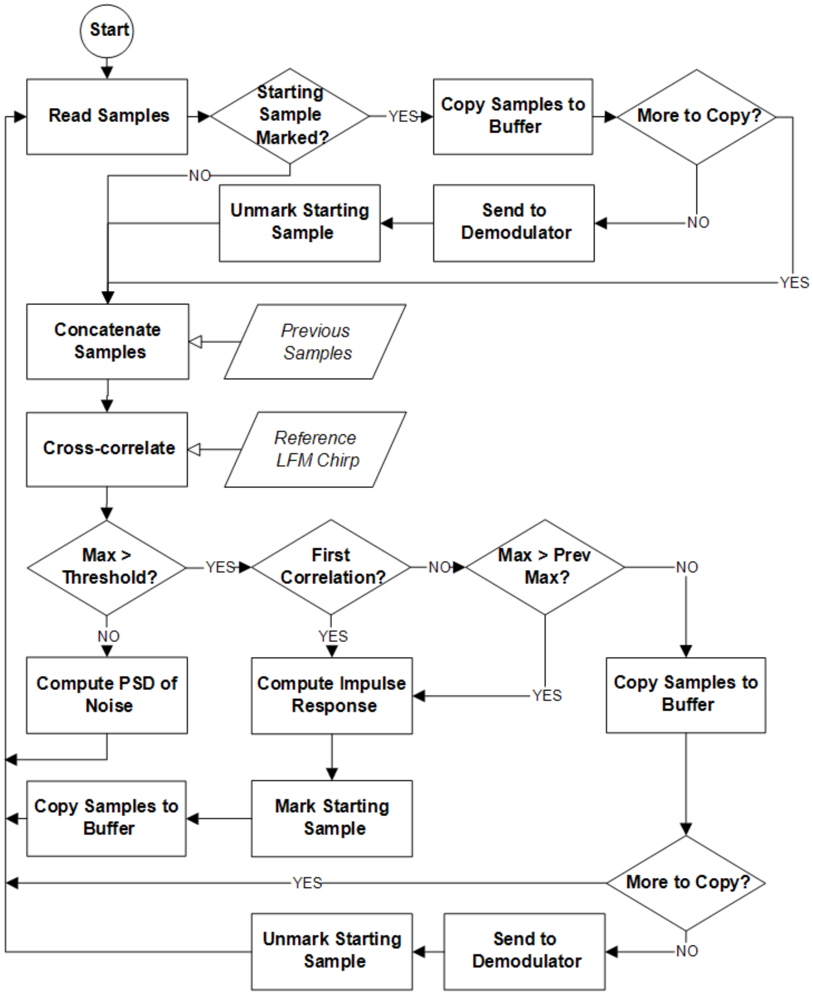
Software architecture of acoustic modem, with arrows depicting the flow of data generated by the network application through the system and down to the sound card.

Modem Architecture

- Transmit and receive tasks are in modular, “assembly line” fashion
- Each stage is a separate thread
- Threads communicate by placing the resulting item on inter-connecting thread-safe queues







Start

Read Samples

Starting Sample Marked?

YES

Copy Samples to Buffer

More to Copy?

NO

Unmark Starting Sample

Send to Demodulator

NO

YES

Concatenate Samples

Previous Samples

Cross-correlate

Reference LFM Chirp

Max > Threshold?

YES

First Correlation?

NO

Max > Prev Max?

NO

NO

Compute PSD of Noise

YES

Compute Impulse Response

YES

Copy Samples to Buffer

Copy Samples to Buffer

Mark Starting Sample

YES

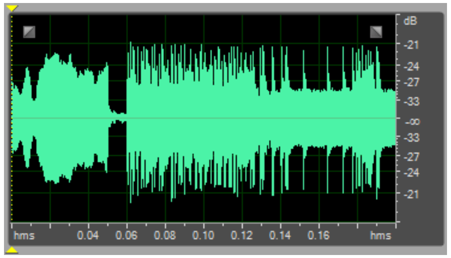
More to Copy?

NO

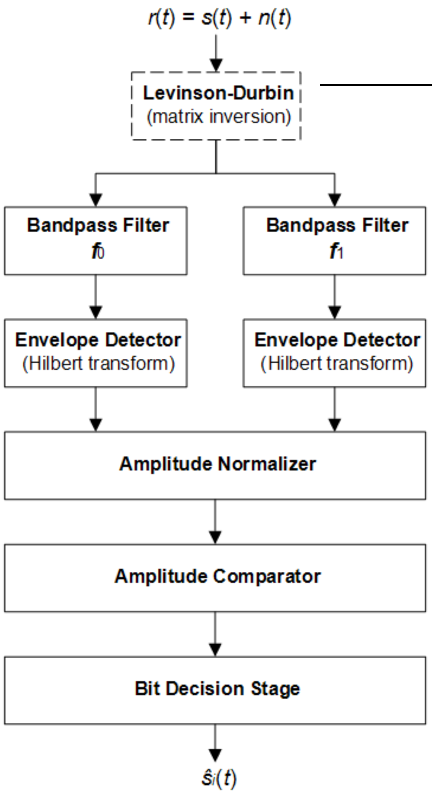
Unmark Starting Sample

Send to Demodulator

Capture/Correlate Block of Receiver



Unequalized data frame



$r(t) = s(t) + n(t)$

Levinson-Durbin (matrix inversion)

Bandpass Filter f_0

Bandpass Filter f_1

Envelope Detector (Hilbert transform)

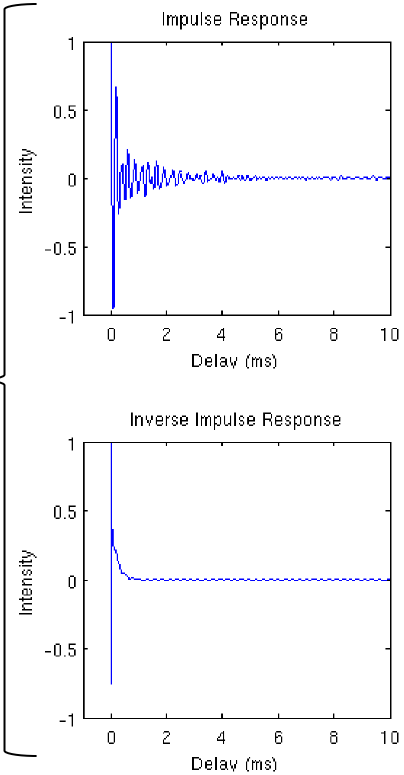
Envelope Detector (Hilbert transform)

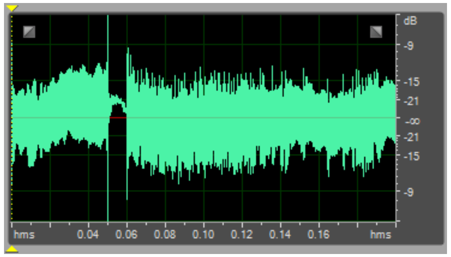
Amplitude Normalizer

Amplitude Comparator

Bit Decision Stage

$\hat{s}_i(t)$





Equalized data frame

Stages of Noncoherent FSK Receiver

Control Interface Parameters

- **CHIRP_MS** = <integer>
- **BASE_FREQUENCY_RX / TX** = <integer>
- **FULL_DUPLEX** = <TRUE/FALSE>
- **GUARD_MS** = <integer>
- **IMPULSE_RISE_MS** = <decimal>
- **INVERSE_FILTER** = <TRUE/FALSE>
- **NUMBER_OF_CARRIERS** = <2/4>
- **PARITY_BYTES** = <integer>
- **PAYLOAD_SIZE_IN_BYTES** = <integer>
- **SYMBOLS_PER_SECOND** = <integer>
- **THRESHOLD** = <integer>

Computational Performance

- Measured with JRat
- Each frame had
  - 50 ms LFM chirp
  - 10 ms guard time
  - 4-byte frame header
  - 16 parity bytes
  - 128 bytes of payload (including other headers)
  - Total of 1184 bits
- Frames transmitted at 1 kbps

Processing Time of Subroutines (ms)

	Desktop Intel Q6600	Laptop T60p Intel T7200	Laptop T500 Intel P8400
Transmit			
a. Modulate	8.00	12.00	13.40
b. Encode Reed-Solomon	9.33	74.40	82.60
Sum (a:b)	17.33	86.40	96.00
Frame duration	1244.00	1244.00	1244.00
Comp Time/Signal Length	1.39 %	6.95 %	7.72 %
Receive			
c. Cross-correlation	2.36	5.03	4.46
Block length	85.33	85.33	85.33
Comp Time/Signal Length	2.77%	5.89%	5.23%
Demodulate			
d. Levinson-Durbin	3.40	3.80	5.33
e. FFT convolution	29.80	65.00	43.83
f. Bandpass filtering	2.60	3.60	4.16
g. Envelope detection	61.60	117.60	84.50
h. Normalizer	1.60	3.70	1.50
i. Comparator	0.40	2.00	0.33
j. Bit Decision	0.40	1.60	0.50
k. Decode Reed-Solomon	1.33	21.40	17.40
l. Write 2 wav files	2.00	3.40	2.60
m. Write IR data to csv file	16.33	55.40	36.00
Sum (d:m)	119.46	277.50	196.15
Frame duration	1244.00	1244.00	1244.00
Comp Time/Signal Length	9.60 %	22.31%	15.77%

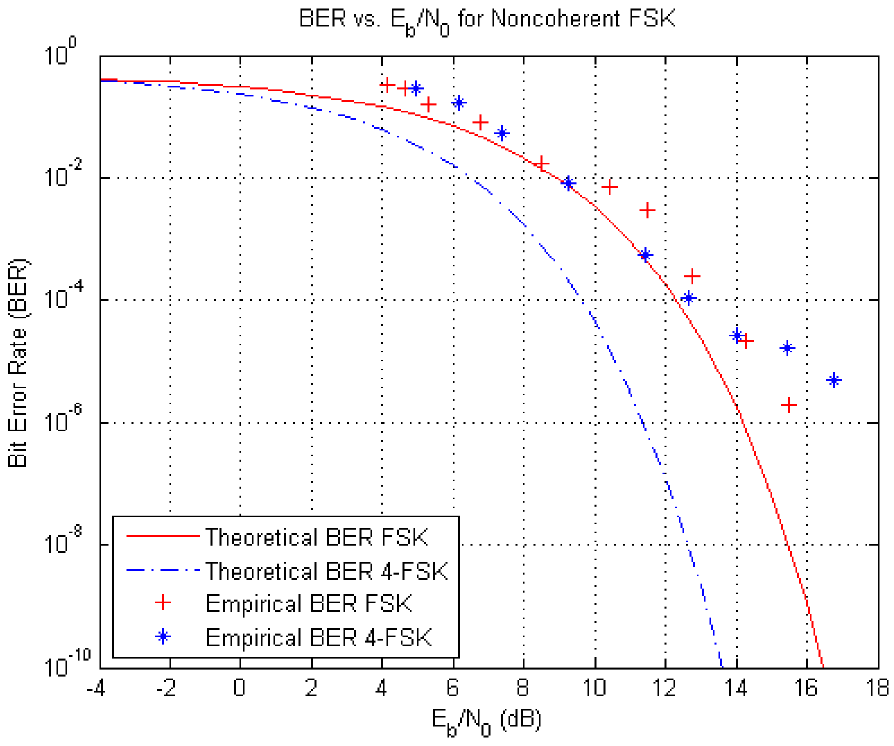
Performance in AWGN Channel

Performance test results for binary FSK

E_b/N_0	BER	Bits Sent	Error Bits
4.13	0.337	103,600	34,926
4.66	0.296	103,600	30,677
5.32	0.162	103,600	16,742
6.76	0.0777	103,600	8,045
8.52	0.0174	103,600	1,807
10.40	0.00717	518,000	3,715
11.51	0.00289	518,000	1,498
12.74	0.000251	518,000	130
14.27	2.220E-5	1,036,000	23
15.47	1.931E-6	1,036,000	2

Performance test results for 4-FSK

E_b/N_0	BER	Bits Sent	Error Bits
4.95	0.286	103,600	29,652
6.16	0.169	103,600	17,492
7.38	0.0533	103,600	5,521
9.25	0.00829	207,200	1,718
11.44	5.598E-4	414,400	232
12.67	1.081E-4	1,036,000	112
14.01	2.684E-5	1,554,000	41
15.43	1.689E-5	2,072,000	35
16.76	4.826E-6	2,072,000	10



Empirical and theoretical BER vs. E_b/N_0.

Summary and Future Work

- Summary
  - Implemented open source acoustic modem
  - Modem offers numerous configuration parameters
  - Performs channel characterization and records data
- Future Work
  - Add other modulation techniques
  - Convert to LMS-based adaptive DFE
  - Deploy a pair of modems for long-term channel characterization and communication experiments

Key Contributions

- Characterization of the Hudson River estuary at 200 and 505 meters
- Channel characterization procedure described in great detail, possibly more thoroughly than any other work
- A network simulation written in OMNeT++ / MATLAB that simulates the underwater acoustic channel and PHY layer of a network stack
- A software-driven binary and 4-FSK modem with a zero-forcing equalizer and Reed-Solomon codes
- Software for the channel characterization, network simulation, and software modem that facilitates converting formulas and block diagrams into working systems

Publications

- Brian Borowski and Dan Duchamp, *Measurement-based Underwater Acoustic Physical Layer Simulation, in Proceedings of MTS/IEEE OCEANS 2010, September 2010, Seattle, Washington (to appear).*
- Brian Borowski and Dan Duchamp, Short Paper: *The Softwater Modem - A Software Modem for Underwater Acoustic Communication, in Proceedings of the ACM International Workshop on Underwater Networks (WUWNet'09), November 2009, Berkeley, California.*
- Brian Borowski, *Characterization of a Very Shallow Water Acoustic Communication Channel, in Proceedings of MTS/IEEE OCEANS 2009, October 2009, Biloxi, Mississippi.*

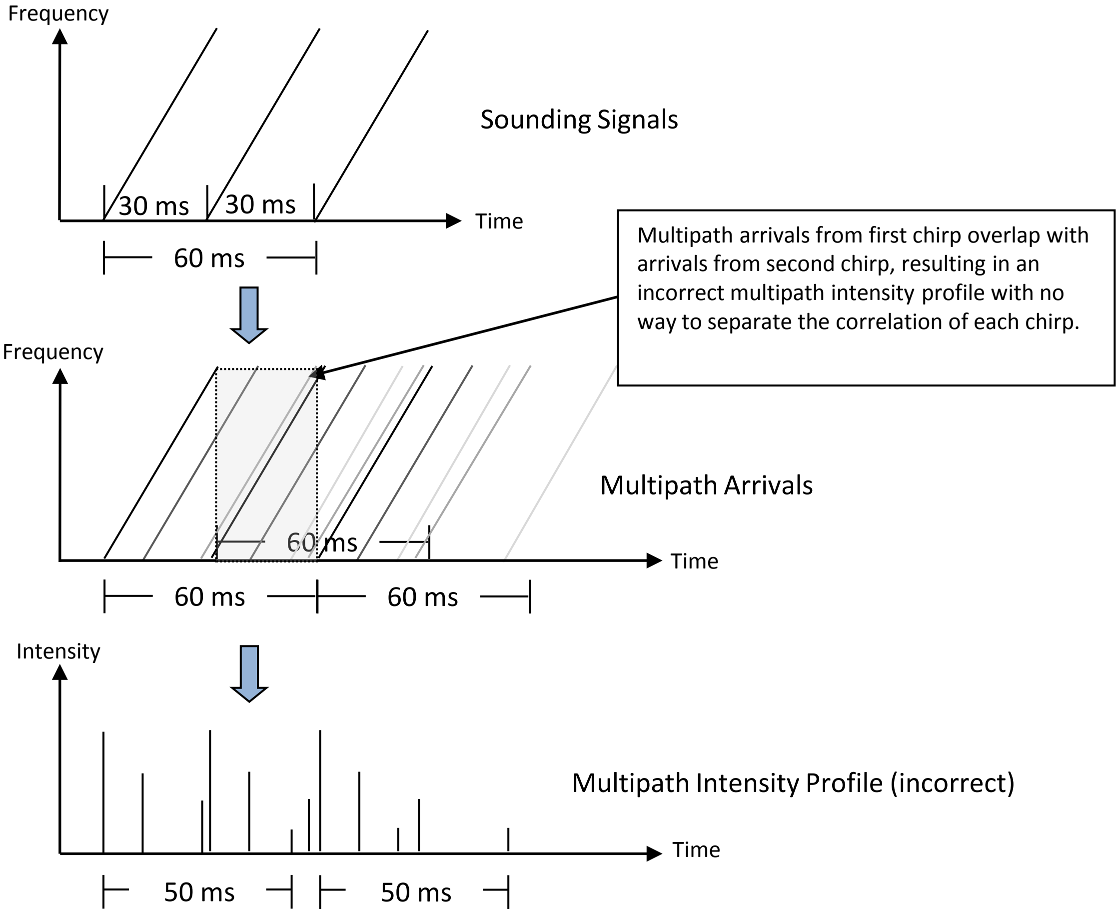
- Brian Borowski, Alexander Sutin, Heui-Seol Roh, and Barry Bunin, *Passive Acoustic Threat Detection in Estuarine Environments, in Proceedings of SPIE Vol. 6945, March 2008, Orlando, Florida.*
- Brian Borowski, Heui-Seol Roh, Barry Bunin, and Alexander Sutin, *Estimation of Passive Acoustic Threat Detection Distances in Estuarine Environments, in Proceedings of the 153rd Meeting of the Acoustical Society of America, June 2007, Salt Lake City, Utah.*

  (Placed second in the Best Student Paper competition of the Engineering Acoustics section)

Any Questions?

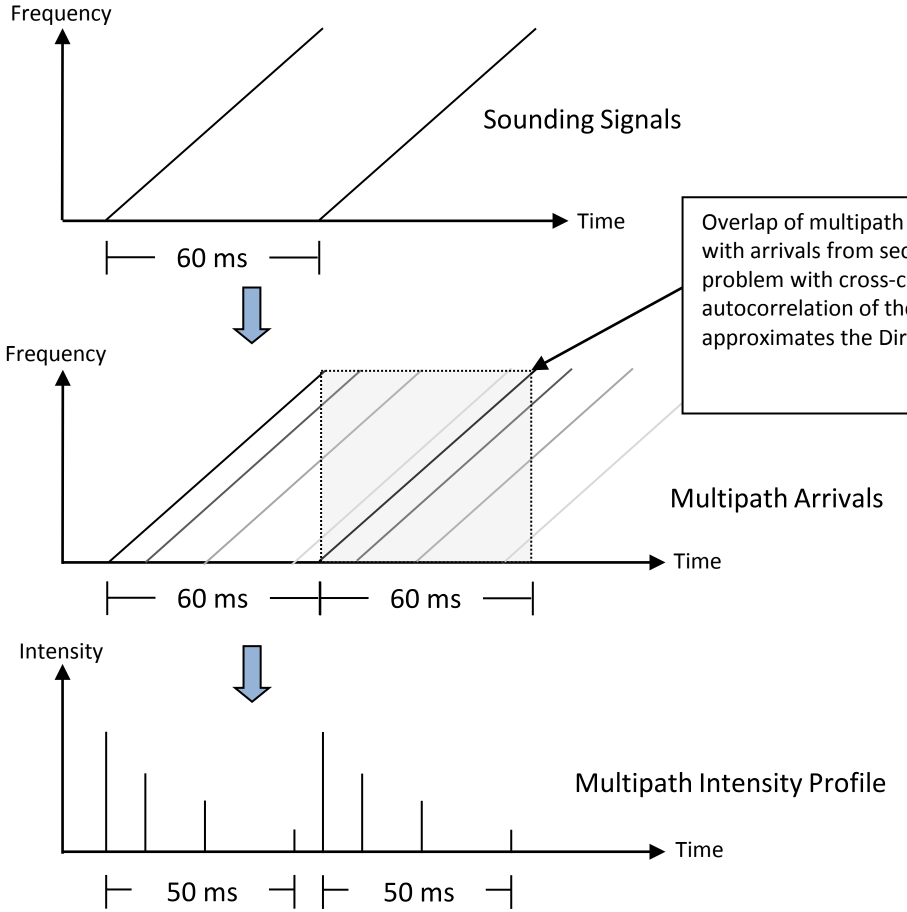
Extra Slides

Incorrect Channel Sounding



Multipath arrivals from first chirp overlap with arrivals from second chirp, resulting in an incorrect multipath intensity profile with no way to separate the correlation of each chirp.

Correct Channel Sounding



Overlap of multipath arrivals from first chirp with arrivals from second chirp produces no problem with cross-correlation as long as the autocorrelation of the sounding signal nearly approximates the Dirac delta function.

Environmental Conditions

- Recorded at the Castle Point Buoy, at 40.74348° latitude and -74.02263° latitude

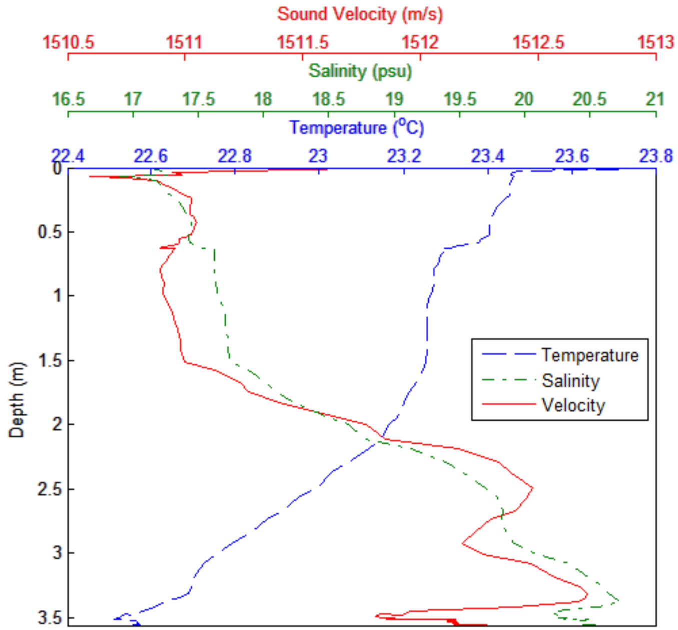
Channel	505 m	200 m
Start Time	5:14 P.M.	6:04 P.M.
Temperature	76°F	75°F
Relative Humidity	55%	57%
Wind Speed	10 knots	8 knots
Wind Direction	159°	159°

Sound Velocity Profile

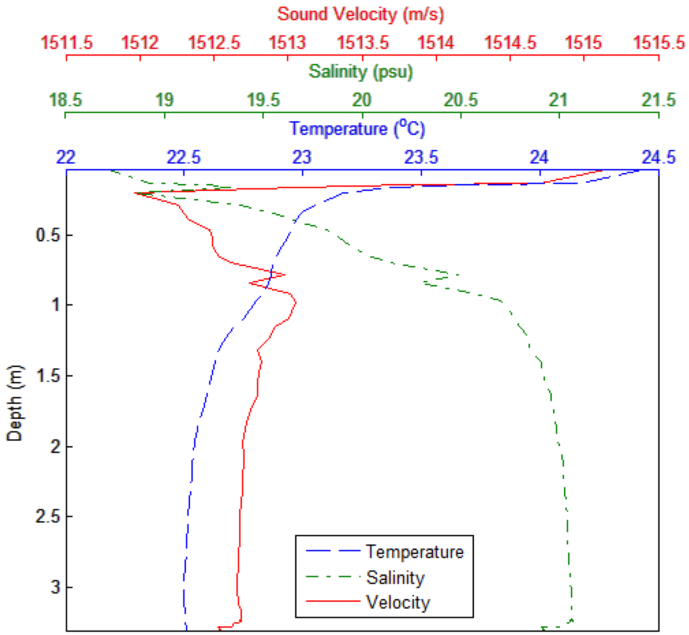
Medwin's expression:

$$c = 1449.2 + 4.6T - 5.5 \times 10^{-2}T^2 + 2.9 \times 10^{-4}T^3 + (1.34 - 10^{-2}T)(S - 35) + 1.6 \times 10^{-2}D$$

Limits:
$0 \leq T \leq 35$°C
$0 \leq S \leq 45$ psu
$0 \leq D \leq 1000$ m



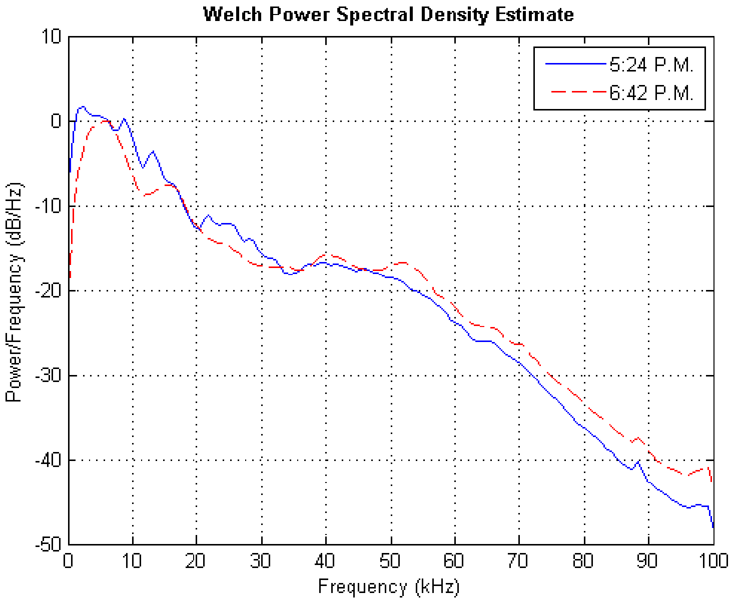
Sound velocity profile for 505-meter channel



Sound velocity profile for 200-meter channel

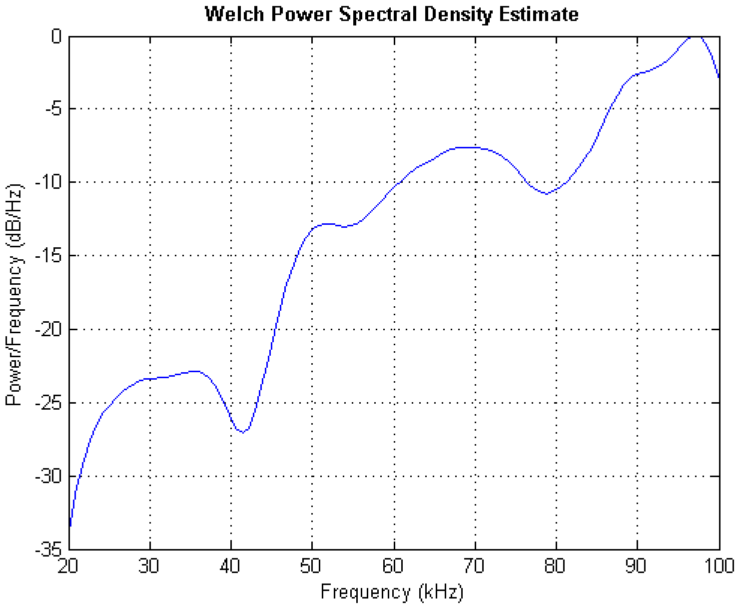
Ambient Noise

- Recorded for 30 seconds before emitting test signals
- Power spectral density (PSD) of noise was estimated via a Welch periodogram technique based on a 256-point FFT together with a Hanning window and no overlap

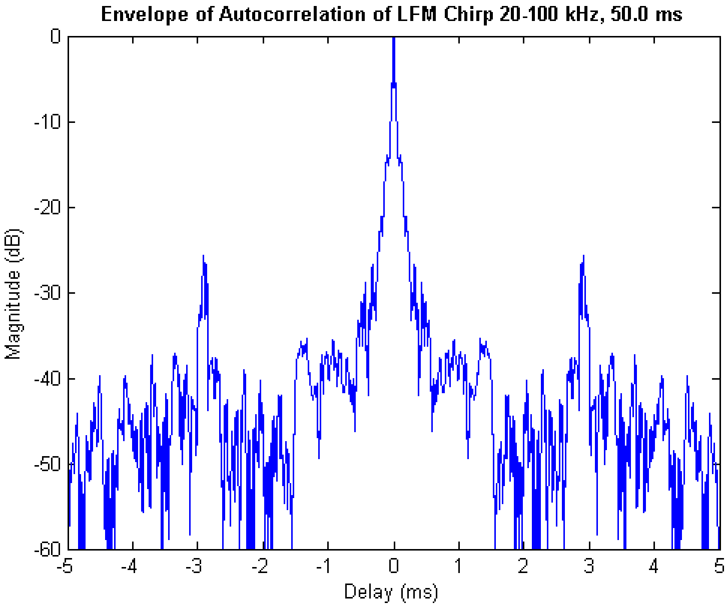


PSD of ambient noise in Hudson River estuary

Signal Properties

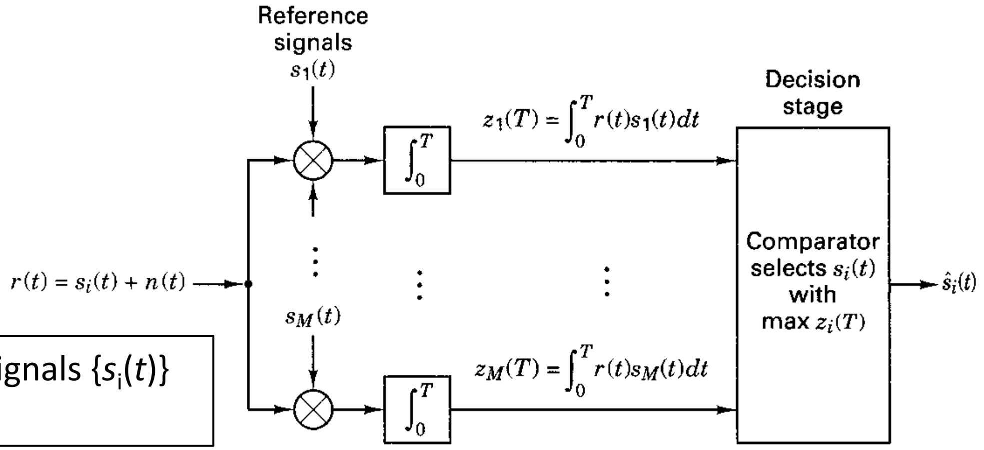


PSD of chirp signal at 1m
(frequency response of emitter)



Envelope of emitted chirp waveform
autocorrelation function

Demodulation



Correlation receiver with *M* reference signals {$s_i(t)$} [Sklar 2001]

```
%% Demodulates PSK signal using a correlation receiver.
t = 0:samplesPerBit-1;
psk0 = -cos(2 * pi * carrierFreq/samplingRate * t);
psk1 =  cos(2 * pi * carrierFreq/samplingRate * t);
rxPSK = zeros(1, numberOfBits);
for i = 1:numberOfBits
    rcv = packet((i-1)*samplesPerBit + 1:i*samplesPerBit);
    zero = rcv .* psk0;
    one = rcv .* psk1;
    z0 = sum(zero);
    z1 = sum(one);
    rxPSK(i) = (z1 > z0);
end
```

MATLAB code implementing a correlation receiver for PSK signals